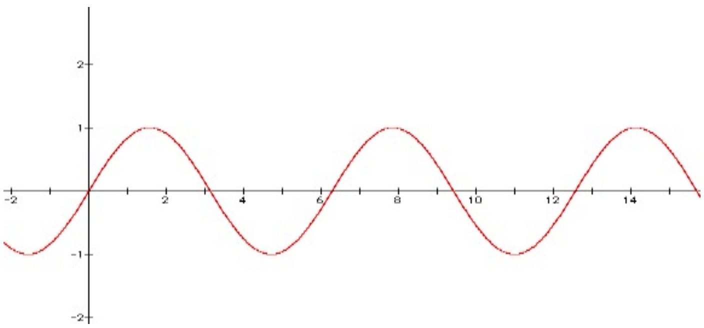
Mathematics Tutorial Series

Differential Calculus

Trigonometric Functions I

The most important fact about trig functions is that they are **periodic**. This means that after some period, the whole graph just repeats.

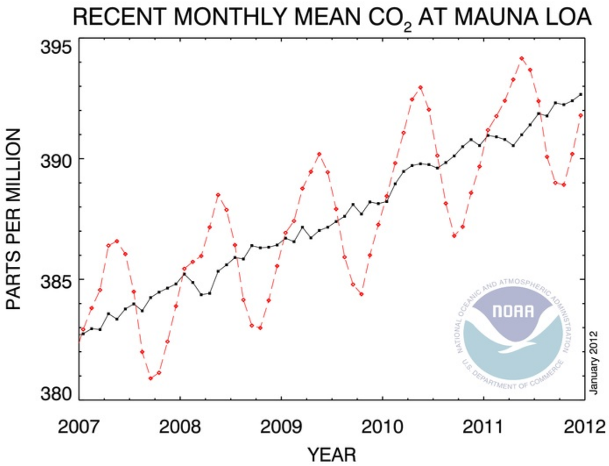


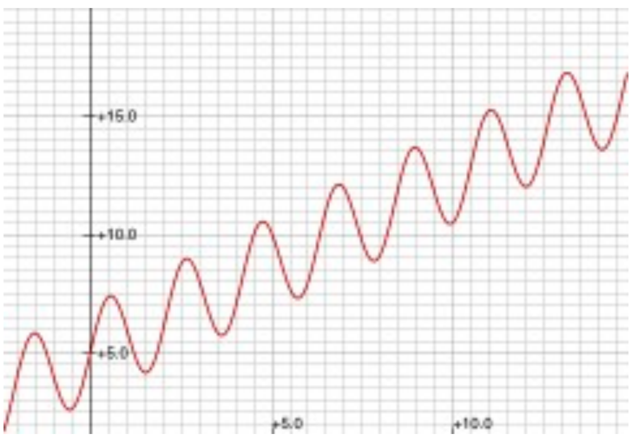
This makes these functions important for any **mathematical model** that tries to describe a periodic phenomenon.

Examples:

1. Any wave phenomenon like radio, microwave, Wi-Fi, elementary particles
2. Blood pressure as the heart beats
3. Orbits and pendulums
4. Tides and tidal power
5. Road traffic flows on a daily cycle
6. Annual heating cost of your home

To model and understand any of these topics, you will have to use trig functions.





$$y = 5 + 0.75x + 2\sin(3x)$$

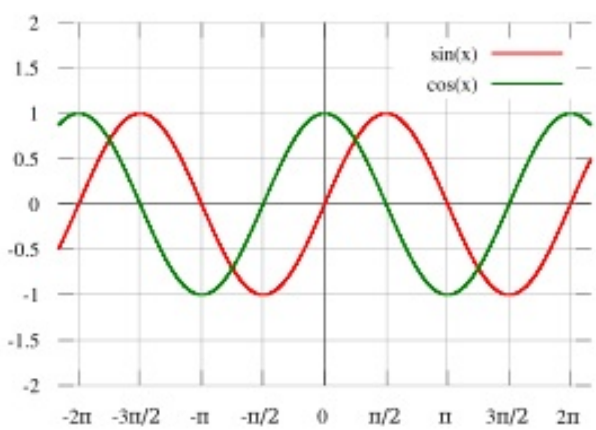
The main functions are sin x, $cosx$ and $tan\ x$.

"sin" is short for "sine" – hence the name.
"cos" is short for "cosine"
"tan" is short for "tangent"

It is customary to measure the variable x in **radians** when doing calculus. The reason is simple. We want the derivative of $sin\ x$ to be $cos\ x$. If we don't work in radians this isn't true.

This means that the period of $sin\ x$ and of $cos\ x$ is 2π.

$\sin 0 = 0$	$\cos 0 = 1$
$\sin \frac{\pi}{2} = 1$	$\cos \frac{\pi}{2} = 0$
$\sin \pi = 0$	$\cos \pi = -1$
$\sin \frac{3\pi}{2} = -1$	$\cos \frac{3\pi}{2} = 0$
$\sin 2\pi = 0$	$\cos 2\pi = 1$



$$\mathbf{-1 \le \sin x \le +1}$$
$$\mathbf{-1 \le \cos x \le +1}$$

How do you calculate a value for sin x?

The Unit Circle

A useful fact about sin and cos is that these functions give the coordinates of the points on the unit circle.

For each angle t the point $(\cos t, \sin t)$ is on the circle

$$x^2 + y^2 = 1.$$

Interactive Mathematics site: www.intmath.com

Sine function demo

Trig Identities – very handy

$$\sin^2 x = (\sin x)(\sin x)$$

$$\cos^2 x + \sin^2 x = 1$$

$$cos(x + h) = cos\, x\, cos\, h - sin\, x\, sin\, h$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin(x + h) = \sin x \cos h + \cos x \sin h$$

$$\sin 2x = 2 \cdot \sin x \cdot \cos x$$

For example, suppose we want $\sin\left(\frac{\pi}{12}\right)$.

Now $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

So $\sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \sin\frac{\pi}{3}\cos\frac{-\pi}{4} + \cos\frac{\pi}{3}\sin\frac{-\pi}{4}$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{-\sqrt{2}}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

Summary

1. Trig functions are periodic
2. In calculus, we work in radians for measuring angles
3. Trig functions are used to model periodic phenomena
4. $\cos^2 x + \sin^2 x = 1$
5. There are a few trig identities that can be useful
6. The functions $\sin x$ and $\cos x$ are periodic with period 2π.
7. For each function, the variable x can take any real value and the value y is in the range $-1 \leq y \leq +1$.
8. Special values: $\sin 0 = 0$ and $\cos 0 = 1$